The Product Rule

Introduction: We already have a property that allows us to find the derivative of a **sum** (or difference) of two nctions.

Example:

If $f(x) = x^3 + x^5$,

then $f'(x) =$ ______ + ______

Property:

If $f(x) = g(x) + h(x)$,

then $f'(x) =$ ______ + ______

Objective: We want to discover a property that can be applied to find the **product** of two functions.

Let $g(x) = x^3$ and $h(x) = x^5$. Now, let $f(x) = g(x) \cdot h(x) =$ ________ • ________ = ________.

Find: $f'(x) =$ ______ $g'(x) =$ ________

$h'(x) =$ ________

$g'(x) \cdot h'(x) =$ __________

So, notice that although it is true that: **If $f(x) = g(x) + h(x)$, then $f'(x) = g'(x) + h'(x)$,**
it is not true that: **If $f(x) = g(x) \cdot h(x)$, then $f'(x) = g'(x) \cdot h'(x)$.**

It is possible to get the correct answer for $f'(x)$ by a clever combination of the equations for $g(x)$, $h(x)$, $g'(x)$, and $h'(x)$.

Write the functions again.

$g(x) =$ ______, $h(x) =$ ______, $f(x) =$ ______, $g'(x) =$ ______, $h'(x) =$ ______, $f'(x) =$ ______

(Remember, we are trying to get the function $f'(x)$ from a combination of $g(x)$, $h(x)$, $g'(x)$, and $h'(x)$.)

See if you can figure out what this combination is (with the following hints)!

Notice that the **8** in $f'(x) = 8x^7$ is the sum of the **3** and **5** in $g'(x) = 3x^2$ and $h'(x) = 5x^4$.

Fill in the following

$$f'(x) = 8x^7 = 5x^7 + 3x^7 = \_\_\_\_\_\_ \cdot 5x^4 + \_\_\_\_\_\_ \cdot 3x^2.$$

Notice what the functions are that you put in the blanks!

Now try to complete the conjecture. This derivative property is called the **Product Rule.**

Product Rule: If $f(x) = g(x) \cdot h(x)$, then $f'(x) =$ ______ · ______ + ______ · ______

When you think you have it, try the following. If you don't have it, work at it!
Assume that your conjecture is true for the product of any two functions.

If $f(x) = f(x) = x^2 \cdot \ln x$, then $f'(x) =$ ______ · ______ + ______ · ______.

= ____________ or ____________

Check your result graphically by graphing f(x) in Y1, your derivative in Y2, and the nDeriv(Y1,X,X) in Y3.

Try one more.

If $f(x) = x^3e^{4x}$, then $f'(x) =$ ________ $\cdot$ ________ $+$ ________ $\cdot$ ________.

$=$ ____________________ or ____________________

Again, check your result graphically.

Examples of the Product Rule:

1. Let $f(x) = (3x^2 + 4)(2x^2 + 3)$

   a. Find f'(x) by using the Product Rule.

   b. Find f'(x) by expanding the terms first, then applying the Power Rule.

   c. Show that the two answers are equivalent.

2. Let $f(x) = x^2(x+3)^2$

   a. Graph f on your calculator.

   b. From the graph of f, for what x-values does it appear that f has a horizontal tangent?

   x= ____________________

   c. Analytically, find all values of x where the graph of f has a horizontal tangent line.